QUADRATIC EQUATIONS (TERM-2)

Type-3 Questions.

(PULKIT JAWAL)

DISCRIMINANT METHOD OR QUADRATIC FORMULA.

Q1. Find the roots of the following quadratic equations by Quadratic formula.

(a) $x^2-4x-21=0$

(b) $5m=6-m^2$

(c) $6=t(1+t)$

(d) $6y^2=2+y$

(e) $1=-100x^2+20x$

(f) $6-x-x^2=0$

(g) $2x-3=(x-2)^2+1$

(h) $40m+3m^2-m^3=0$

(i) $x^2-\frac{5x}{2}+\frac{3}{2}=0$

(j) $\frac{-5}{m}=-2-\frac{2}{m^2}$

(k) $24m^2-41m+12=0$

(L) $-231x-11x^2+22x^3=0$

(m) $-12-m^2-5\sqrt{3}m=0$

(n) $2x^2-2\sqrt{6}x+3=0$

(o) $11m=-6\sqrt{3}-m^2\sqrt{3}$

(p) $x^2+1.5=2.5x$

(q) $x(2x+1)=6$

(r) $3x^2-2\sqrt{6}x+2=0$

Q2. Find the roots of the equation $x(3\sqrt{7}x+4)=\sqrt{7}$ by discriminant Method.

Q3. Find the roots of the equation $\sqrt{2}=-x^2+x+\sqrt{2}x$ by Quadratic formula.

Q4. Find the roots of $36x^2-12ax+(a^2-b^2)=0$ by Quadratic formula.

Q5. Find the roots of $50\sqrt{3}x^2+550x+300\sqrt{3}=0$.

Q6. Find the roots of $-13x=2\sqrt{6}-x^2 4\sqrt{6}$ by Quadratic formula

Q7. Find the roots of (a) $\frac{1}{(x-3)}+\frac{1}{(x+5)}=\frac{1}{3}$ (b) $\frac{1}{x+4}=\frac{11}{30}+\frac{1}{x-7}$

Q8. Find the roots of the equation $4x^2-4a^2x+(a^4-b^4)=0$ by Quadratic formula

Q9. Find the roots of the following equation.

(a) $a^2b^2x^2+b^2x-a^2x-1=0$ (b) $-41m=-12-24m^2$ (c) $40+8x-x^2=0$

Q10. Find the roots of equation. $x^3=x^2\sqrt{3}+\sqrt{3}x+x^2$

Q11. Find the roots of equation $x^2+m^2x^2+2mcx+c^2-a^2=0$

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Typ-2

SOLUTIONS

Q1

(a) $x^2-4x-21=0$

on comparing with $ax^2+bx+c=0$

we have $a=1, b=-4, c=-21$

Now $D=b^2-4ac=(-4)^2-4(1)(-21)$

$D=16+84=100$

$\alpha=\frac{-b+\sqrt{D}}{2a}=\frac{-(-4)+\sqrt{100}}{2\times1}$

$\alpha=\frac{4+10}{2}=\frac{14}{2}=7$

$\beta=\frac{-b-\sqrt{D}}{2a}=\frac{4-10}{2}=\frac{-6}{2}=-3.$

Roots are $x=7,-3$

(b) $5m=6-m^2$ OR $m^2+5m-6=0$.

$a=1, b=5$ & $c=-6$.

$\alpha=\frac{-b+\sqrt{D}}{2a}=\frac{-5+\sqrt{5^2-4(1)-6}}{2\times1}$

$\alpha=\frac{-5+\sqrt{25+24}}{2}=\frac{-5+\sqrt{49}}{2}$

$\alpha=\frac{-5+7}{2}=1$

Similarly $\beta=\frac{-5-7}{2}=\frac{-12}{2}=-6.$

$\therefore$ Roots are $(1,-6)$

(c) $6=t(1+t)$ OR $t^2+t-6=0$

$t=2,-3$

α β.

(d) $6y^2=2+y$ OR $6y^2-y-2=0$

$y=\frac{2}{3}$ & $y=\frac{-1}{2}$

α β.

(e) $1=-100x^2+20x$ OR $100x^2-20x+1=0$

$x=\frac{1}{10},\frac{1}{10}$

α β.

(f) $6-x-x^2=0$ OR $x^2+x-6=0$

$x=2,-3$

(g) $(2x-3)=(x-2)^2+1$

$\Rightarrow x^2-6x+8=0$

$x=2,4$

(h) $40m+8m^2-m^3=0$

$\Rightarrow m(40+8m-m^2)=0$

$\Rightarrow -m^2+8m+40=0$

$\Rightarrow m^2-8m-40=0.$

$\Rightarrow m=-5,8$

(i) $x^2-\frac{5x}{2}+\frac{3}{2}=0$

$2x^2-5x+3=0$

$a=2, b=-5, c=3.$

$D=1$

$\alpha=\frac{5+\sqrt{1}}{4}=\frac{5+1}{4}=\frac{6}{4}$

$\alpha=3/2$ $\beta=1$

OR $x=\frac{3}{2}$ & $x=1$

(j) $\frac{-5}{m}=-2-\frac{2}{m^2}$

$\Rightarrow -5m=-2m^2-2$

$\Rightarrow 2m^2+2-5m=0$

$2m^2-5m+2=0$

$m=\frac{1}{2},2$

(k) $24m^2-41m+12=0$

$a=24, b=-41, c=12$

$D=(41)^2-4(24)(12)=529$

$\sqrt{D}=\sqrt{529}=23.$

$\alpha=\frac{41+23}{2\times24}=\frac{4}{3}$

& $\beta=3/8$

(l) $-231x-11x^2+22x^3=0$

$-11(21x-x^2+2x^3)=0$

$-11x(21-x+2x^2)=0.$

$\Rightarrow 2x^2-x+21=0$

$x=\frac{7}{2},-3$

(m) $-12-m^2-5\sqrt{3}m=0.$

$\Rightarrow 12+m^2+5\sqrt{3}m=0$

$\Rightarrow m^2+5\sqrt{3}m+12=0$

$m=-\sqrt{3},-4\sqrt{3}.$

(n) $2x^2-2\sqrt{6}x+3=0$

$a=2, b=-2\sqrt{6}, c=3.$

$D=(-2\sqrt{6})^2-4(2)(3)$

$D=24-24=0$

$\alpha=\frac{2\sqrt{6}+\sqrt{0}}{2\times2}=\frac{2\sqrt{6}}{2\times2}=\frac{\sqrt{6}}{2}$

$\beta=\frac{2\sqrt{6}-\sqrt{0}}{4}=\frac{\sqrt{6}}{2}$

(o) $11m=-6\sqrt{3}-m^2\sqrt{3}.$

$\Rightarrow m^2\sqrt{3}+11m+6\sqrt{3}=0$

$a=\sqrt{3}, b=11, c=6\sqrt{3}$

$D=(11)^2-4(\sqrt{3})(6\sqrt{3})=121-72=49.$

$\alpha=\frac{-2}{\sqrt{3}}$, $\beta=\frac{-9}{\sqrt{3}}$ (OR) $\alpha=\frac{-2\sqrt{3}}{3}$, $\beta=-3\sqrt{3}$

(p) $x^2+1.5-2.5x=0$

$x^2-2.5x+1.5=0.$

$x=\frac{3}{2},1$

(q) $x(2x+1)=6$

$2x^2+x=6$

$\Rightarrow 2x^2+x-6=0.$

$\Rightarrow x=\frac{3}{2},-2$

(r) $3x^2-2\sqrt{6}x+2=0.$

$x=\frac{\sqrt{6}}{3}$ $\frac{\sqrt{6}}{3}$.

Q2. $x(3\sqrt{7}x+4)=\sqrt{7}$

$\Rightarrow 3\sqrt{7}x^2+4x=\sqrt{7}$

$\Rightarrow 3\sqrt{7}x^2+4x-\sqrt{7}=0$

$a=3\sqrt{7}, b=4, c=-\sqrt{7}$

$D=(4)^2-4(3\sqrt{7})(-\sqrt{7})=16+21\times4$

$D=16+84=100.$

$\alpha=\frac{-4+\sqrt{100}}{2\times3\sqrt{7}}=\frac{-4+10}{2\times3\sqrt{7}}=\frac{6}{2\times3\sqrt{7}}=\frac{1}{\sqrt{7}}$

$\beta=\frac{-4-10}{2\times3\sqrt{7}}=\frac{-14}{2\times3\sqrt{7}}=\frac{-7}{3\sqrt{7}}$

Q3. $\sqrt{2}=-x^2+x+\sqrt{2}x$

$\Rightarrow \sqrt{2}=-x^2+x(1+\sqrt{2})$

$\Rightarrow x^2-x(1+\sqrt{2})+\sqrt{2}=0$

$a=1, b=-(1+\sqrt{2}), c=\sqrt{2}$

$D=(-(1+\sqrt{2}))^2-4(1)(\sqrt{2})$

$D=(1+\sqrt{2})^2-4\sqrt{2} \Rightarrow 1+2+2\sqrt{2}-4\sqrt{2}$

$D=3-2\sqrt{2}$

$\alpha=\frac{(1+\sqrt{2})+\sqrt{3-2\sqrt{2}}}{2\times1}$, $\beta=\frac{(1+\sqrt{2})-\sqrt{3-2\sqrt{2}}}{2\times1}$.

Type-3 (Quadratic Equations)

(Solutions)

Q4. $36x^2 - 12ax + (a^2 - b^2) = 0$

$a = 36,\ b = -12a,\ c = (a^2 - b^2)$

$D = 144a^2 - 144(a^2 - b^2)$

$D = 144a^2 - 144a^2 + 144b^2 = \boxed{144b^2}$

$\boxed{\sqrt{D} = 12b.}$

$\alpha = \dfrac{12a + 12b}{2 \times 36} = \boxed{\dfrac{a+b}{6}}$

$\boxed{\beta = \dfrac{a-b}{6}}$

Q5. $50\sqrt{3}x^2 + 550x + 800\sqrt{3} = 0.$

$50(\sqrt{3}x^2 + 11x + 6\sqrt{3}) = 0$

$\sqrt{3}x^2 + 11x + 6\sqrt{3} = 0.$

$D = (11)^2 - 4(\sqrt{3})(\sqrt{3})$

$D = 121 - 24 \times 3 = 121 - 72$

$D = 49 \quad \therefore \boxed{\sqrt{D} = 7}$

$\alpha = \dfrac{-11 + 7}{2\sqrt{3}} = \dfrac{-4}{2\sqrt{3}} = \boxed{\dfrac{-2}{\sqrt{3}}}$

$\beta = \dfrac{-11 - 7}{2\sqrt{3}} = \dfrac{-18}{2\sqrt{3}} = \boxed{\dfrac{-9}{\sqrt{3}}}$

Q6. $-13x = 2\sqrt{6} - x^2 4\sqrt{6}.$

$\Rightarrow x^2 4\sqrt{6} - 13x - 2\sqrt{6} = 0$

$a = 4\sqrt{6},\ b = -13,\ c = -2\sqrt{6}.$

$D = b^2 - 4ac = (-13)^2 - 4(4\sqrt{6})(-2\sqrt{6})$

$\Rightarrow 169 + 192 = 361$

$\sqrt{D} = 19.$

$\alpha = \dfrac{13 + 19}{2 \times 4\sqrt{6}} = \dfrac{32}{2 \times 4\sqrt{6}} = \boxed{\dfrac{4}{\sqrt{6}} \text{ OR } \dfrac{2}{3}\sqrt{6}.}$

$\beta = \dfrac{13 - 19}{2 \times 4\sqrt{6}} = \dfrac{-6}{2 \times 4\sqrt{6}} = \boxed{\dfrac{-3}{4\sqrt{6}} \text{ OR } \dfrac{-\sqrt{6}}{8}}$

Q7. (a) $\dfrac{1}{(x-8)} + \dfrac{1}{(x+5)} = \dfrac{1}{3}$

On solving we get $x^2 - 4x - 21 = 0$

$a = 1,\ b = -4,\ c = -21$

$D = 100, \quad \sqrt{D} = 10.$

$\alpha = \dfrac{4 + \sqrt{D}}{2 \times 1} = \dfrac{4 + 10}{2} = \dfrac{14}{2} = 7$

$\beta = \dfrac{4 - 10}{2} = \dfrac{-6}{2} = -3.$

$\therefore \boxed{7 \text{ and } -3}$

(b) $\dfrac{1}{(x+4)} = \dfrac{11}{30} + \dfrac{1}{x-7}$

On solving we get

$x^2 - 3x - 12 = 0$

$D = 1, \quad \sqrt{D} = 1$

$\alpha = \dfrac{3+1}{2} = \dfrac{4}{2} = \boxed{2}$

$\beta = \dfrac{3-1}{2} = \boxed{1}$

Q8. $4x^2 - 4a^2x + (a^4 - b^4) = 0$

$a = 4,\ b = -4a^2,\ c = (a^4 - b^4)$

$D = (-4a^2)^2 - 4(4)(a^4 - b^4)$

$D = 16a^4 - 16a^4 + 16b^4$

$\sqrt{D} = 4b^2$

$\alpha = \dfrac{4a^2 + 4b^2}{2 \times 4} = \boxed{\dfrac{a^2 + b^2}{2}}$

$\beta = \boxed{\dfrac{a^2 - b^2}{2}}$

Q9. (a) $a^2b^2x^2 + b^2x - a^2x + 1 - b^2 = 0$

$(a^2b^2 - a^2)x^2 + x(b^2 - a^2) - 1 = 0$

$a = (ab)^2 - a^2,\ b = (b^2 - a^2),\ c = -1.$

$D = (b^2 - a^2)^2 - 4a^2b^2(-1)$

$\Rightarrow (b^2 - a^2) + 4a^2b^2$

$\Rightarrow b^4 + a^4 - 2a^2b^2 + 4a^2b^2$

$\Rightarrow (b^2)^2 + (a^2)^2 + 2a^2b^2$

$\Rightarrow (b^2 + a^2)^2$

$\therefore \sqrt{D} = (b^2 + a^2)$

$\alpha = \dfrac{-(b^2 - a^2) + b^2 + a^2}{2a^2b^2}$

$\Rightarrow \dfrac{-b^2 + a^2 + b^2 + a^2}{2a^2b^2} = \dfrac{2a^2}{2a^2b^2}$

$\boxed{\alpha = \dfrac{1}{b^2}}$

$\beta = \dfrac{-b^2 + a^2 - b^2 - a^2}{2a^2b^2} = \dfrac{-2b^2}{2a^2b^2} = \boxed{\dfrac{-1}{a^2}}$

(b) $-41m = -12 - 24m^2$

$\Rightarrow 24m^2 - 41m + 12 = 0$

On solving $\boxed{\alpha = 3/8,\ \beta = 4/3.}$

(c) $40 + 3x - x^2 \geq 0$

$-x^2 + 3x + 40 \geq 0$

$x^2 - 3x - 40 = 0.$

$\therefore \boxed{x = -5, 8}$

Q10. $x^3 = x^2\sqrt{3} + \sqrt{3}x + x^2$

$x^3 - x^2\sqrt{3} - \sqrt{3}x - x^2 = 0.$

$x(x^2 - x\sqrt{3} - \sqrt{3} - x) = 0.$

$\Rightarrow x^2 + x - x\sqrt{3} - \sqrt{3} = 0$

$\Rightarrow \boxed{x^2 - x(1 + \sqrt{3}) - \sqrt{3} = 0.}$

$\Rightarrow$ On solving we get

$\boxed{x = \sqrt{3},\ 1}$

Q11. $x^2 + m^2x^2 + 2mcx + (c^2 - a^2) = 0$

$\Rightarrow (1 + m^2)x^2 + (2mc)x + (c^2 - a^2) = 0.$

$a = (1 + m^2),\ b = 2mc,\ c = (c^2 - a^2)$

$D = (2mc)^2 - 4(1 + m^2)(c^2 - a^2)$

$D = 4m^2c^2 - 4(c^2 - a^2 + m^2c^2 - m^2a^2)$

$D = 4m^2c^2 - 4c^2 + 4a^2 - 4m^2c^2 + 4m^2a^2$

$D = 4m^2a^2 + 4a^2 - 4c^2$

$D = 4(m^2a^2 + a^2 - c^2)$

$\sqrt{D} = 2\sqrt{m^2a^2 + a^2 - c^2}$

$\alpha = \dfrac{-2mc + 2\sqrt{m^2a^2 + a^2 - c^2}}{2(1 + m^2)}$

$\beta = \dfrac{-2mc - 2\sqrt{m^2a^2 + a^2 - c^2}}{2(1 + m^2)}$

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